Γ is therefore the complex difference between reflection coefficients of cryoload and horn, etc.

The problem of coherent multiple reflections on low-noise antennas was predicted by Silver [2], and is familiar to radioastronomers [3]. Since such reflections result from the addition of complex-voltage reflection coefficients, considerable attention has to be paid to such problems as coupler directivity, circulator isolation, and inherently well-matched waveguide sections and antennas. In the case of the radiometer of Hardy *et al.* a −20-dB cross coupler should have at least 20 dB of directivity, or a waveguide isolator should have at least 20 dB of isolation. They would, of course, have to be integrated into the temperature-controlled enclosure, and would somewhat increase the mass. Another possibility would be multiprobe coupling into the waveguide, or noise injection in the coaxial line, following an input isolator. In any event, caution is necessary when employing low-directivity noise injection in sensitive radiometers.

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Computer Program Descriptions

The ZEPLS Program for Solving Characteristic Equations of Electromagnetic Structures

PURPOSE: The program calculates the zeros of an analytic function in particular zones of the complex plane, inside the region where the function is holomorphic and single valued.

LANGUAGE: Fortran V for the Univac 1110 computer.

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AVAILABILITY: ASIS/NAPS Document No. 02551.

DESCRIPTION: Many electromagnetic problems concerning either open or closed structures lead to the equation

$$f(z) = 0 \tag{1}$$

where z is a complex variable and $f(z)$ an analytic function, generally many valued, sometimes having polar singularities.

The program presented in this description is based on the well-known formula which gives the Nth-order moments of the n_z zeros of a holomorphic function within a closed region D

$$s_N = \sum_{l=1}^{n_z} z_l^N = \frac{1}{2\pi j}\int_{+\partial D} z^N \frac{f'(z)}{f(z)}\,dz,$$

$$\text{if } f(z) \neq 0,\ \forall\, z \in \partial D. \tag{2}$$

Although the use of rectangles would appear to be a more logical choice, for better accuracy the most suitable shape of D was found to be a circle, which can be deprived of some internal circles. Such circles allow the exclusion of regions where $f(z)$ is not single-valued and/or is not holomorphic. The integral in (2) has therefore to be calculated as the sum of the integrals along the n_c circumferences constituting the contour of D. Each integral has been calculated numerically by means of the formula of Delves and Lyness [1]. Once the moments s_N are known, the zeros $z_l (l = 1,2,\cdots, n_z)$ can be calculated by solving an algebraic equation of degree n_z.

During execution the program goes through the following steps:

1) The examined function, which has to be defined through a specific routine, is evaluated along the n_c circumferences in the n_{si} points chosen for the ith circumference. It is possible that some zeros (in number of $n_{zk} \geq 0$) may be previously known and given among the input variables. In any case, the number n_{zu} of unknown zeros must be less than 5. During this first step, program execution is interrupted and returned to the calling program in the following cases: a) the number of unknown zeros is either $=0$ or ≥ 5; in the latter case it is necessary to reduce the size of D; b) the variation of the function is too rapid between consecutive points along the ith circumference; in such a case it becomes impossible to determine the number n_z of the zeros in D and it is necessary to increase n_{si}; c) either a zero or a pole is recognized along one of the circumferences.

2) At the second step, the integrals which give the moments s_N are evaluated.

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TABLE I

Fortran name	Dimension	Type	Variable	Meaning	Input/Output
NC	-	I	n_c	Number of circles constituting the contour of D	Input
ZO	(2,NC)	DP	z_{oi}	Coordinates of the circle centres ($i=1,2,\ldots n_c$). The first circle is the external one.	Input
R	(NC)	DP	r_i	Radii of the circles ($i=1,2,\ldots n_c$).	Input
NS	(NC)	I	n_{si}	Number of points where the function has to be evaluated ($i=1,2,\ldots n_c$).	Input
NZK	-	I	n_{zk}	Number of known zeros in D.	Input
ZK	(2,NZK)	DP	z_{kj}	Coordinates of the known zeros ($j=1,2,\ldots n_{zk}$).	Input
NZU	-	I	n_{zu}	Number of unknown zeros.	Output
ZER	(2,4)	DP	z_h	Coordinates of the unknown zeros ($h=1,2,\ldots n_{zu}$)	Output
FMIN	-	DP	$\|f(z)\|_{min}$	Minimum value of the amplitude of f(z) along ∂D.	Output
IPRINT	-	I	-	Controls the printing: if =1, centres and radii of the circles are printed; if =2, also the calculated values of f(z) along ∂D are printed; no printing if =0.	Input
FUN	-	-	-	Name of the subroutine defining f(z).	-
H	(2,L)	DP	-		
Z	(2,L)	DP	-	Working matrices. L must be greater or equal to	
S	(2,4,NC)	DP	-	the sum of n_{si}:	
IS	(L)	I	-	$L \geq \sum_{i=1}^{n_c} n_{si}$	
NT	(NC)	I	-		

3) Finally, the n_{zu} unknown zeros of $f(z)$ in D are calculated by solving an algebraic equation of degree n_{zu}.

The user may call the subroutine from his own program as follows: CALL ZEPLS (NC, ZO, R, NS, NZK, ZK, NZU, ZER, FMIN, IPRINT, FUN, $m, $n, H, Z, S, IS, NT).

The variables in the argument list are given in Table I.

In the event that no solution is obtained, control is returned to the calling program to statement n when the number of unknown zeros is either $=0$ or ≥ 5; to statement m in the other cases.

As is shown in the preceding table, for all the noninteger variables double precision is used. Complex variables are in fact represented by vectors with two elements, containing the real and imaginary parts of the complex number, respectively. Similarly, vectors of complex variables are represented by two-dimension matrices. For instance: ZER(1,3) is the real part of the third unknown zero, and ZER (2,3) is its imaginary part.

As a final check, it is convenient to calculate $f(z)$ at the found zeros, and to compare these values with $|f(z)|_{min}$.

COMMENTS

The precision which can be obtained is essentially influenced by the variation rate of the examined function along the contour of region D and consequently by the number of points taken on the contour. It is therefore clear that a zero or a pole close to the contour results in decreased accuracy. In such a circumstance there are two ways out: either to increase the number of points or to modify the region D. In any case, it should be borne in mind that on increasing the number of points, the accuracy reaches a maximum. The value n_s, for which this maximum is obtained, depends upon the function and upon the region position; for higher values of n_s the accuracy decreases because of the rounding error.

The storage occupation of the ZEPLS routine is 8338 decimal words; this figure does not take into account the storage which is necessary for dimensioning vectors and matrices defined in the calling program.

The program has been used to find the eigenvalues of the modes of propagation in a waveguide partially filled with magnetized ferrite. The results were in perfect agreement with the values calculated by Barzilai and Gerosa [2], [3].

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